Complex Networks: From the U.S. Congress to U.S. College Football

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Outline

- "Complex networks"
- Communities in networks
- NCAA Division-IA Football
 - Rankings from biased random walks
- United States Congress
 - Committee assignment network
 - Quantifying the politics of Representatives and committees
 - Legislation cosponsorship and roll call voting networks
- Facebook networks and other current projects
- Summary

General References

- Survey/review articles
 - S. H. Strogatz [2001], "Exploring Complex Networks," *Nature* **410**, 268-276.
 - M. E. J. Newman [2003], "The Structure and Function of Complex Networks," SIAM Review 45(2), 167-256.
- Netwiki: http://netwiki.amath.unc.edu/











Community Structure



Concepts and buzzwords: Hierarchical clustering, graph partitioning, betweenness, modularity, local vs. global methods

From leaves to root...

- 1) Start without connections
- 2) Identify connection with strongest weight
- 3) Connect
- 4) Check to see if any components merged
- 5) Return to Step 2

Fewer options for unweighted networks, as it is unclear how to start this process...

From root to leaves...

- Identify weakest connection/edge (e.g., by weight or betweenness)
- 2) Remove
- 3) Check to see if component breaks
- 4) Return to Step 1

Different ways to identify "strength," depending on size of network and whether it is weighted or unweighted

Recent theory: Eigenvector-based modularity maximization of M. E. J. Newman, *PNAS/PRE* 2006.

College Football

- T. Callaghan, P. J. Mucha, & MAP [2004], "The Bowl Championship Series: A mathematical review," *Notices of the AMS* **51**, 887-893.
- TC, PJM, & MAP [2007], "Random walker ranking for NCAA Division I-A football," *American Mathematical Monthly* **114**(9), 761-777.



http://rankings.amath.unc.edu/

Disclaimer



ESPN The Magazine



NCAA Division-IA Football

- Teams (nodes) connected to each other by games played (edges)
- In 2005, the 119 Division I-A teams played a total of 690 games prior to end-of-season bowl games
- Diameter = 4
- Single connected component in 3-4 weeks
- Most teams play majority of games inside their own conferences (ACC, SEC, etc.)
- One of the only sports at any level that doesn't determine champions in a playoff



Community Structure



 Strong conference structure in Div-IA

Girvan-Newman betweenness-based algorithm (PNAS, 2002), counting geodesics through each edge, clearly identifies different conferences

Biased Random Walk on Graph



Random-Walker Rankings

- Randomly select a single game played by your "favorite" team
- 2) Flip weighted coin (heads with prob. p)
- 3) Heads: go with winner; tails: go with loser
- 4) Return to Step 1

An individual random walker will never settle down, but an ensemble has well-defined steady-state statistics Interesting mathematics in the asymptotics for different value of p and in round-robin tournaments.

2007 Rankings (10/27/07)

Random walkers (p = 0.75)

BCS (now called FBS)

- 1. Arizona State
- 2. Boston College
- 3. LSU
- 4. Oregon
- 5. Kansas
- 6. Ohio State
- 7. Georgia (13th for BCS)
- 8. West Virginia
- 9. Oklahoma
- 10. Connecticut (15th for BCS)

- 1. Ohio State
- 2. LSU
- 3. Arizona State
- 4. Oregon
- 5. Boston College
- 6. Kansas
- 7. West Virginia
- 8. Oklahoma
- 9. South Florida (11th for us)
- 10. Missouri (14th for us)

Rankings & Communities



Changing the outcome of a high betweenness edge/game (interconference) typically affects rankings more than doing so to a lower betweenness game (intraconference)

Congress: A Popular American Villain

 "It could be probably be shown by facts and figures that there is no distinctly American criminal class except Congress."

-- Mark Twain

 "Suppose you were an idiot and suppose you were a member of Congress. But I repeat myself."

-- Mark Twain



Congressional Committee Assignment Networks



Committees and th subcommittees connected by the Representatives through committee assignments.

Weights assigned v either (a) raw interl of common membe or (b) normalized interlock in terms o expected overlap.

Congressional Committees Assignments

- AMS Mathematical Moment: "Unearthing Power Lines"
- MAP, P. J. Mucha, M. E. J. Newman, & C. M. Warmbrand [2005] "A network analysis of committees in the U.S. House of Representatives," *Proc. Nat. Acad. Sci.* **102**, 7057-62.
- MAP, A. J. Friend, PJM, & MEJN [2006], "Community structure in the U.S. House of Representatives," *Chaos*, 16(4), 041106.
- MAP, PJM, MEJN, & AJF [2007], "Community structure in the United States House of Representatives," *Physica A* 386(1), 414-438.

Committee Assignment Network

- Bipartite graph of 115-165 committees and about 440
 Representatives and Delegates assigned to committees.
- Typical Representative sits on 2 Standing or Select committees, and about 2 subcommittees of each.
- Much of detailed work in making U.S. law occurs in committees and subcommittees.
- Network is dense relative to many popular examples (good warmup for phylogenetics).
- Major recent changes:
 - 1994 elections ("Republican Revolution")
 - 9/11 and Homeland Security

108th House







Quantifying Politics

- Voting matrix of roll call, +1/-1 (Representatives vs. measures)
- Singular value decomposition (SVD) identifies that most of the variance of the votes is in first two modes (eigenvectors) [see Poole & Rosenthal, Sirovich]
- First mode ~ "Partisanship"
- Second mode ~ "Bipartisanship"



107th House



Legislation Cosponsorship Network

- Two Congressmen are connected if they sponsor/cosponsor legislation
- "Higher dimensional" data than committee assignments
 - Can be seen using modularity maximization
- Shows that polarization in Congress was gradual rather than abrupt
 - Can be quantified using modularity
- Y. Zhang, AJF, A. L. Traud, MAP, J. H. Fowler, PJM, submitted to *Physica A* (arXiv: 0708.1191)

108th Senate (colored by party)



108th House (colored by party)



108th House (colored by state)



108th House (colored by DW-Nominate)



Partisanship via modularity

 Strong rank correlation: DW-Nominate versus components of leading modularity eigenvector





Partisanship via modularity

- Modularity at first leadingeigenvector split (good approximation of maximum) up sharply in early 1990s in both houses of Congress
- Modularity obtained when partitioning by party lines also up sharply and becomes closer to that given by eigenvector
- Increased polarization in Congress appears in bill cosponsorship (and roll call)



Political realignments via modularity



- A. Waugh, L. Pei, ALT, MAP, JFH, & PJM, in preparation.
 - Note: being sent to a political science journal...
- Uses roll call voting data
- Future work: voting in UK parliament (need students/postdoc

Facebook

- Some community detection results (a tutorial with Facebook as working example)
 - ALT, E. Kelsic, PJM, & MAP, in preparation
- Friendship network among college students
- Data for 100 schools
- Different structures from different network growth mechanisms?
 - Olga Mandelshtam, Summer 2007
 - Need students/postdocs!



Caltech network

Current and Future Work

- Comparison of different Congressional networks
 - Committee/subcommittee assignments, legislation cosponsorship, roll call votes
 - Note: committee data available on request
- Some generalizations on eigenvector community detection for three-way splittings (UNC students)
- U.S. Supreme Court precendent network (anyone?)
- Baseball Hall of Fame rankings (anyone?)
- Baseball pitcher rankings (anyone?)
- Network growth mechanisms with Facebook and Supreme Court networks (anyone?)
- UK voting networks (anyone?)
- Always trying to acquire other interesting data...
- I'm actively trying to recruit students and postdocs...

Summary

- Investigate processes on networks (football team rankings, Congressional collaborations, collegiate social life, etc.) by studying hierarchical structure
- Development and focus: novel data
- Undergraduate students leading or involved
- Reprints and preprints: <u>http://www.maths.ox.ac.uk/~porterm</u> or <u>porterm@maths.ox.ac.uk</u>
- Netwiki: http://www.netwiki.amath.unc.edu
 - Wiki for network science